When does the genetic relationship between relatives end?

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Abstract

A simple argument shows that for kinships farther away than 6 genetic steps the genetic relation between blood relatives becomes negligible. This holds, among other examples, for third cousins and beyond, and for 7th degree ancestors.

1. Genetic relation between blood relatives

The genetic relation between two individuals A and B is defined by the number of chromosomes that those individuals have in common. To simplify the problem we:

1. ignore the effect of mutation: so we assume that the chromosomes be invariant in time.
2. neglect the difference in genetic structure between women and men in the pair of sex chromosomes
3. neglect effects of inbreeding
4. assume that the chromosomes of non-related persons are different.

For the moment I consider direct offspring. In the next section I consider cousins etc.

The genetic relation of a father with each of his children is 1/2, as a child shares (exactly) half of his father's chromosomes. Similarly, the relation of a grandfather with his grandchild is 1/4, as 1/4 of the grandchild's chromosomes are those of his grandfather[1].

So:

person A:	himself	son	grandson	great-grandson	greatgreat-grandson	etc
steps from A:	0	1	2	3	4	...
relation	$(½)^0$=1	$(½)^1$=1/2	$(½)^2$=1/4	$(½)^3$=1/8	$(½)^4$=1/16	...

So if *n* is **the number of genetic steps**[2] that separate two individuals, then the genetic relation *Rel* (and hence the fraction *Fract* of shared chromosomes) is:

$$Fract = Rel = \left(\frac{1}{2}\right)^n \qquad (1)$$

[1] Note that in the latter case, the number of ¼ shared chromosomes represents an expectation value: it may happen that a given grandchild possesses more than 25% of his grandfather's chromosomes, or less than 25%. For offspring that is more than one generation away, the division of the chromosomes from that ancestor is a random process with an expectation value that depends on the distance from that ancestor.

[2] In this simple case (an ancestor and his offspring), the number of genetic steps *n* between these persons is equal to their **degree of kinship**: father/son have 1st degree of kinship, grandfather/grandson 2nd degree of kinship, etc.

The quantity *Rel* can run from one to zero, where zero stand for completely unrelated – in the definition of Eq(1) there is no limit on kinship between remote relatives, as there remains always a residual value of *Rel*. For instance, a 10^{th} degree grandchild (10 generations down) has a genetic relation of 1/1024 – small, but still not zero.

However, there is a practical limit to the number of shared chromosomes, as a human being has only 46 chromosomes in his nuclei. The number of chromosomes N_{shared} that an ancestor shares with his n^{th} degree grandson is:

$$N_{shared} = 46 \times Fract = 46 \times \left(\tfrac{1}{2}\right)^n \qquad (2)$$

So, for a 10^{th}-degree grandson (n=10) the number of shared chromosomes is 0.05 – way below unity. Fractional chromosomes do not exist. So, the real interpretation of this value 0.05 is that these persons have a probability of 5% that they share one single chromosome.

The discrete number of chromosomes provides a means to define the limit of (detectable) genetic relationship. A criterion that makes sense is to assume that the genetic basis of kinship disappears when there is less than 50% probability that the two persons share one single chromosome. This puts *Fract* on a value of 0.5/46 -- which can for practical purposes be rounded off to 1/100.

From Eq. (1) it then follows that the genetic relationship becomes insignificant when the number of genetic steps between two persons *n* is larger than $^{2}\log 100 = 6.6$

So, in case of **7^{th} degree ancestors** or **7^{th} degree offspring**, the degree of genetic relation has become biologically meaningless. This implies that one would never recognize a family relationship from portraits of an ancestor as remote as the 7^{th} degree or beyond.

2. Genetic relation with brothers, cousins etc.

The calculation of the number of shared chromosomes (and hence of *Rel viz. Fract*) for brother, cousins etc. is somewhat more complicated by the fact that these persons are usually connected by more than one genetic string.
For instance, for two half-brothers A and B via a father one has one string, consisting of two genetic steps:
A – father – B
However, for two full brother one has two of such strings:
A – father – B
A – mother – B

It can be proven (see e.g. Hartl, 1980) that *Rel* can be calculated by making up Eq (1) for all strings and then by adding them up:

$$Fract = Rel = \sum_{all.strings} \left(\frac{1}{2}\right)^n \qquad (3)$$

For the string A-father-B, as well as for the string A-mother-B, *n* = 2. Therefore for half-brothers *Rel* is $(½)^2$=1/4 (hence the same as for grandfather/grandson). But for full brothers, *Rel* = *Rel (via mother)* + *Rel (via father)* = $(½)^2 + (½)^2 = 1/2$ – which is same value as for father/son[3].

The degree of kinship is the number of genetic steps n_{eff} that would be required if two persons were connected by one single string instead of by multiple strings. It can be calculated from *Rel* by:

Degree of kinship = $n_{eff} = {}^2\log(1/Rel)$ (4)

Examples of *Rel* and degrees of kinship

	Rel	degree of kinship
a person with respect to himself (or with respect to a clone); identical twins	1	0
father/son; full brothers	1/2	1
half-brothers; uncle/cousin; grandfather/grandsons	1/4	2
first cousins[a]; great-grandfather	1/8	3
first cousins[b]; 4th degree ancestor	1/16	4
half-brother via father; first cousin via mothers[†]	3/8	1.4
second cousins[a]; 5th degree ancestor	1/32	5
second cousins[b]; 2nd cousins[a] once removed*; 6th degree ancestor		6
third cousins[a]; 7th degree ancestor	1/128	7

[a]via full brothers/sisters (so sharing two ancestors)
[b]via half-brothers or half-sisters (so sharing one ancestor)
[†]may happen when a man marries successively with two (full) sisters
*so: children of your second cousin

From the criterion above (*Rel*< *1/100*) it follows that the genetic relationship becomes insignificant when the degree of kinship two persons n_{eff} is larger than $^2\log 100 = 6.6$

This implies that in case of **third cousins** the degree of genetic relation has become biologically meaningless.
Similarly, in case of **7th ancestors**, the degree of genetic relation has become biologically meaningless

Reversely, according to this criterion, your 6th degree ancestor still may have a biologically significant genetic relation with you. It is interesting to relate that such an ancestor is a person from two centuries ago.

3. Postscript

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[3] If a string happen to pass a pair of identical twin brothers or sisters T_1 and T_2, the number of steps in that string should be subtracted by one. Example: 1st cousins A, B via identical twin brothers are connected via A- T_1-grandfather- T_2-B (normally 4 steps, now 3 steps) and via A- T_1-grandmother- T_2-B (also 3 steps). So *Rel* = $(½)^3 + (½)^3 = 1/4$ – hence 1st cousins via identical twins are as closely related as half-brothers are.

It should be noted that the number of chromosomes that is present in a given society is finite. This means that individuals in such a society have always a certain genetic relationship with each other. How large this relationship is, depends on the society and on the interaction with other societies. Blood relation in such societies has only a meaning if the relationships exceed the ‘average’. I do not know what the ‘average’ relationship in e.g. Germany is, but I would say that the level of 7th kinship is not a bad estimate. If so, my estimation of 6.6 for the value of the degree of kinship where genetic relationship becomes meaningless, may well apply to societies like Germany. Of course, it is much too low for small isolated societies maintaining a long tradition of intermarriage.

References:

Hartl, Daniel L. (1980), *Principles of Population Genetics*, Sinauer Associates Inc., Sunderland Mass, ISBN 0-87893-272-0. [A good book on genetic relations]

Wright, Sewall G. (1922), *Coefficients of inbreeding and relationschip*, American Naturalist **56**, 330-338 [the founding paper of the subject]