PERIODICITIES OF ECLIPSES

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In a recent article, Smiley (1975) discusses the value of a statement of van den Bergh (1955) on periodicities of solar or lunar eclipses. This statement is:

> "The chief result to be inferred from our study will prove to be: every period, every distance in time between any two eclipses (both either solar or lunar, of course) is built up of either a combination of a whole number of Inex and a whole number of Saros, or a whole number of Saros or a whole number of Inex exclusively."

Thus, every time interval between any two eclipses can be expressed as a linear combination of Saros (S, 223 lunations) and Inex (I, 358 lunations) with integral coefficients. Smiley showed that *one* single lunation (van den Bergh's (1955) 'nova') equals $38I - 61S$. Multiplying this expression by any integer k, he concluded that the time interval between a solar eclipse and *any* new moon (eclipse syzygy or not) can also be expressed as a linear combination of S and I with integral coefficients. From this, Smiley concludes that van den Bergh's statement is not important at all, and is not very useful for finding other solar eclipses if one is given. According to Smiley, van den Bergh's law is equivalent to the trivial statement that a solar eclipse occurs at new moon.

To resolve this contradiction, we consider a time interval T between two eclipses. Expressing T in lunations, T will always be an integer. According to van den Bergh, two integers m and n may be found, so that

$$T = mS + nI.$$

However, this solution is not unique. Between the Saros (223 lunations) and the Inex (358 lunations), there exists the trivial relation

$$358S - 223I = 0.$$

Using this equality, the complete solution for T reads

$$T = (m + k.358)S + (n - k.233)I$$

where k is an arbitrary integer. Thus, there always exists a *set* of solutions. For instance, the Mayan Thix period (317 lunations) may be expressed as $4I - 5S$ as well as $227I - 363S$.

There is, however, a difference between these representations. After one Saros interval, the eclipse takes place at the same node. This new eclipse is almost identical with the first one; the mean position of the moon with respect to the node has only shifted $-0\overset{\circ}{.}477$ in longitude. After one Inex, the eclipse is also reproduced with great exactness, but at the other node. In this case, the shift is $+0\overset{\circ}{.}041$. So, the shift of $4I$ minus the shift of $5S$ amounts to $2\overset{\circ}{.}6$, which is the shift of the Thix period. If, however, the Thix is represented by $227I - 363S$, the accumulated shift of $227I$ minus $363S$ is $182\overset{\circ}{.}6$, which differs just 180° from the former result. From this it follows that in the latter case the shift is measured from the 'wrong' node, where no eclipse takes place. This is also clear from the following argument. In the expression $4I - 5S$ the number of Inex is even, which means that the eclipse takes place from the same node. But in $227I - 363S$ this number is odd, which means indeed that the shift is measured with respect to the other node, thus differing just 180° from the former result. In fact, adding k times $358S - 223I$ to a given eclipse periodicity is equivalent to saying that by adding $k.180°$ to the longitude of a given node, a new one will be found.

In our example, if one represents the Thix by $4I - 5S$, clearly the shift is minimized. In that case it is possible to construct a series of eclipses, each separated by either one Saros or one Inex, so that the first one is at time zero and the last one is at time $4I - 5S$. A similar statement can be made for any other eclipse periodicity. This is in fact the important property on which van den Bergh founded his prediction technique.

To demonstrate this property, we group some lunar eclipses in a 'panorama' (Table I), in which the horizontal separation is one Inex and the vertical separation is one Saros.

TABLE I

ILLUSTRATIVE PANORAMA OF LUNAR ECLIPSES

1888 Jul 23	1917 Jul 04	1946 Jun 14	1975 May 25	*2004 May 04*
1906 Aug 04	1935 Jul 16	1964 Jun 24	1993 Jun 04	2022 May 16
1924 Aug 14	1953 Jul 26	1982 Jul 06	2011 Jun 15	2040 May 26
1942 Aug 26	1971 Aug 06	2000 Jul 16	2029 Jun 26	2058 Jun 06
1960 Sep 05	1989 Aug 17	2018 Jul 27	2047 Jul 07	2076 Jun 17
1978 Sep 18	2007 Aug 28	2036 Aug 07	2065 Jul 17	2094 Jun 28

Because of the small shifts associated with the Saros and Inex, there is little difference in the magnitude of these eclipses; in this example every lunar eclipse is a total one. The eclipses of 1978 and 2004 are separated by $4I - 5S$, thus by the Thix interval. From this panorama it is easy to construct a series of lunar eclipses connecting the 1978 and 2004 events, for instance:

1978 $(-S)$ 1960 $(+I)$ 1989 $(-S)$ 1971 $(+I)$ 2000 $(-S)$
1982 $(+I)$ 2011 $(-S)$ 1993 $(+I)$ 2022 $(-S)$ 2004.

In this example the series has ten terms, which is obviously the shortest possible series which can connect two eclipses separated by the Thix interval. It is clear, however, that there are many other ways of connecting these eclipses by a series of ten or more terms, especially if one takes into account the fact that Table I can be extended in any direction.

It can easily be shown that for any other representation in which the shift is not minimized, such a series of real eclipses does not exist. Consider, for example, the representation of the Thix by $227I - 363S$, for which the total shift is 182°6. If one tries to construct such a series, this value of the total shift has to be reached by successively adding the shift resulting from one Saros or one Inex. However, for each 'jump' of one Saros or Inex, representing a new term of the series, the total shift changes by 0°477 at most. This means that for many terms, the shift with respect to the first eclipse has to be near 90° in order to reach the final value of 182°6. In these cases, however, there will be no eclipse at all since the moon is obviously too far from a node. So, in these representations, the required series of eclipses does not exist in reality; some of the terms of the connecting series will always be simple full moons.

In such a representation, the expression $T = mS + nI$ is not very helpful for the prediction of eclipses. This is the case for the Thix, if represented by $227I - 363S$, and also for Smiley's counter-example. For eclipse prediction, the integers m and n have to be chosen in such a way that the total shift is minimized. In practice this means that the minimum value of $|m|$ has to be chosen to obtain the proper result. If this is done, van den Bergh's statement is indeed very useful for finding other eclipses if one is given, as is shown in his second book (van den Bergh 1954).

Perhaps the best way to avoid misinterpretations of van den Bergh's law is to restate it as follows:

> "Every period, every distance in time between any two eclipses (both either solar or lunar, of course) can be built up uniquely by a whole number of Inex and a whole number of Saros, or a whole number of Saros or a whole number of Inex exclusively, in such a way that these eclipses are connected by at least one series of eclipses which are all separated by either one single Inex or one single Saros."

References

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van den Bergh, G. 1955, *Periodicity and Variation in Solar (and Lunar) Eclipses*, Tjeenk Willink, Haarlem, Netherlands, pp. 24, 28, table 7.