Triple Conjunctions: Twins and Triplets

G. P. KÖNNEN* and J. MEEUS†
***Royal Netherlands Meteorological Institute, De Bilt, The Netherlands**
†Heuvestraat 31, B-3071 Erps-Kwerps, Belgium

Triple conjunctions between the bright outer planets are presented for the years AD 0 to 3000. It is shown that for Mars–Jupiter and Mars–Saturn, two subsequent triple conjunctions are possible with a time separation of one synodic period.

Triple conjunctions of other outer planets as well as triple conjunctions of the bright outer planets with first-magnitude zodiacal stars are discussed for AD 1900 to 2050.

INTRODUCTION

Although the physical relevance of triple conjunctions (three subsequent conjunctions of two celestial bodies in the course of a year) is nowadays very restricted, periodically these phenomena receive some attention. The reason for this is that triple conjunctions (TCs) may be quite spectacular, so that when a TC of two bright planets or a TC of a bright planet and a bright star takes place, people become interested in future and past occurrences. Such a TC is thus commonly followed by some papers on the subject.

For instance, the triple conjunction of Jupiter and Saturn in 1940–41 gave rise to a study of Heilmann[1] and the recent TC of Mars and Jupiter and of Jupiter and Saturn in 1979–80 and 1980–81, respectively, to a paper by Porter[2]. Similarly, the TCs of Mars–Spica in 1967, Jupiter–Regulus in 1968 and Mars–Antares in 1969 inspired us to study triple conjunctions in three papers (in Dutch, however)[3–5], and one of us (J.M.) more recently published a discussion on TCs of Jupiter–Saturn[6]. In the present paper we summarize the most important results of these papers as well as of Porter's paper, adding further comments but, at the same time, restricting ourselves mainly to the bright outer planets.

GENERAL CONSIDERATIONS

The definition of a triple conjunction depends on the choice of the co-ordinates. A TC may, for instance, take place in right ascension (RA) or in longitude. In most cases, both will happen, but there are exceptions. In 1982, for example, a triple conjunction of Saturn and Spica took place in RA (on Jan. 8, Feb. 25 and Sept. 25), but only one conjunction in longitude (on Oct. 5). This situation is illustrated schematically in figure 1. From the figure it is clear that the probability for this is highest for wide TC in the region of the sky where the ecliptic is close to the equator.

In this paper we shall refer only to conjunctions in longitude, since the plane of the ecliptic seems to be a more useful reference for planetary phenomena than the plane of the Earth's equator. Table I summarizes some main parameters of TC. Here, Δt represents the extreme difference between the dates of oppositions of the planets, where a TC remains possible (denoted as $T_1 - T_2$ by Porter[2]). The long periodic cycle which represents the shortest possible time between two subsequent TCs is denoted by $T_{\min}$. This table is

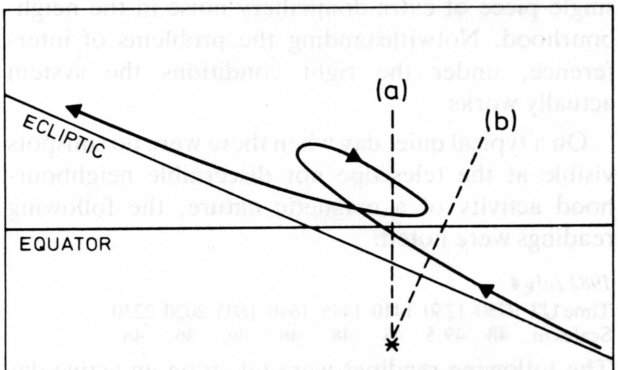

Figure 1. In some cases a triple conjunction takes place in right ascension, while a triple conjunction in longitude is absent. Also, the reverse may happen. The probability for this is highest for wide conjunctions in the regions where the ecliptic is far from the equator.

Table I

Some parameters for triple conjunctions of the bright outer planets

Planets	Δt	$T_{\min}$
Mars–Jupiter	3·9 days	47·0 yr
Mars–Saturn	5·4 days	34·2 yr
Jupiter–Saturn	1·7 days	19·7 yr

Note: Δt denotes the extreme difference between the dates of opposition where a triple conjunction is possible. $T_{\min}$ represents the shortest time interval between two subsequent triple conjunctions. In this table, the eccentricities of the planetary orbits are neglected.

similar to Porter's Table III, and all parameters are calculated for circular orbits.

TWINS AND TRIPLETS FOR MARS–JUPITER AND MARS–SATURN

In Porter's paper a significant approximation was applied, for he did not take into account the influence of the eccentricity of the planetary orbits. In the case of TC of Jupiter–Saturn this approximation is justified, since most of his conclusions remain almost the same. When Mars is involved, however, one arrives at different results.

The influence of the eccentricity on TC has three important effects. Firstly, the quantity Δt will be a function of the opposition date (Δt will be large if the opposition takes place near aphelion). Secondly, the time between two subsequent oppositions varies with the opposition date; this time is relatively small if the opposition takes place near aphelion. Because of these two factors, a TC is relatively easily realized near aphelion. Thirdly, a periodicity for TC can only exist when the number of synodic revolutions of the bodies *and* the number of sidereal revolutions of the Earth are almost integers. This implies that periodicities of TC are commonly longer than expected from circular orbits alone.

Let us consider the second point more closely. After about two years Mars will be in opposition again. The precise time interval may vary between 764 and 811 days, depending on the planet's initial position with respect to its perihelion. For Jupiter, the time between three oppositions is about two years and two months; more exactly between 786 and 806 days. This means, that under favourable conditions, *two* TCs may be produced with a time interval of only two years. In this case, even a third TC is possible after another two years. Obviously, such 'twins' or 'triplets' will only happen if the oppositions take place not far from the perihelion of Mars, and also if the longitudes of the perihelia of Mars and Jupiter differ by only 38°.

For Mars–Saturn a similar situation is possible. At the longitude of the aphelion of Mars (where its movement is slowest), Saturn lies approximately in between its perihelion and aphelion. The minimum time difference between two oppositions of Mars amounts to 764 days, while three oppositions of Saturn are separated by about 757 days in this region of the ecliptic. The difference between these numbers is 7 days. Since $2\Delta t = 13{\cdot}5$ days in this case, a twin is possible. A triplet is not, however.

The question arises, of course, whether twin or triplet periodicities exist. To discuss this, we can take the example of Mars–Jupiter. Here, for circular orbits, a TC may arise every 47 years (see Table I). For every such period, however, the longitude of the heliocentric conjunction increases by about 9°·2. Therefore, after about 1850 years, a TC will take place in the same region of the ecliptic. More exactly, for twins or triplets, there is a periodicity of 1815 years. By similar reasoning, one finds 442 years in the case of Mars–Saturn.

Table II summarizes the main parameters for TCs of Mars–Jupiter and of Mars–Saturn for eccentric orbits. The maximum and minimum values for Δt are given, while the table also contains the periodicities, which are denoted by *TT*.

Table II

Some parameters for triple conjunctions of Mars–Jupiter and Mars–Saturn

Planets	Δt	T_{min}	*TT*
Mars–Jupiter	1·5–5·1 days	2·2 yr	1815 yr
Mars–Saturn	2·8–6·8 days	2·1 yr	442 yr

Note: Here the effect of the eccentricity of the orbits is taken into account. This results in a variation of Δt along the orbits; the extreme values of Δt are presented. Δt reaches its minimum value near the perihelion of Mars. T_{min} denotes the shortest possible time interval between two subsequent triple conjunctions; this equals the synodic period of the planets. *TT* represents a periodicity of triple conjunctions. After such a periodicity a conjunction is repeated with great precision.

Table III lists all 'multiplets' of Mars–Jupiter from 1000 BC to AD 3000; for each conjunction, Δt is given. Note the sharp variation of Δt near the perihelion of Mars. As can be seen from the table, no triplets are found during this period, but there are four twins. Apparently, in some cases, two twins may happen with a time interval of 47 years (as happened in the first millenium BC).

As can be inferred from the data of 880–884, a triplet may, in fact, happen. If the longitude of Jupiter had been 3° less in these years, the time differences between the opposition dates would have been −0·7, +0·0 and +1·1 days (instead of −3·7, −3·0 and −1·9 days respectively) and a triplet would have occurred. Similarly, subtracting one or two degrees from the difference in opposition dates for 2742 to 2746, one finds also that a case of yes–no–yes may occur. However, this does not happen between 1000 BC and AD 3000, since the table shows only 'normal twins'. The periodicity of 1815 can be clearly seen from the table.

Twins of Mars–Saturn are given in Table IV; these happen near the aphelion of Mars because more oppositions take place here than near its perihelion. This, combined with the fact that Δt is relatively large near the aphelion, results in relatively many twins. Indeed, between AD 0 and 3000, seven twins occur.

TRIPLE CONJUNCTIONS OF THE BRIGHT OUTER PLANETS

Table IV shows all triple conjunctions in longitude of the bright outer planets for AD 0 to 3000. The Jupiter–

Table III

List of all triple conjunctions in longitude of Mars–Jupiter separated by two years ('twins'), 1000 BC to AD 3000

Year	*Opposition date of Mars* (ET)	*Opposition date of Jupiter* (ET)	*Difference* (days)	Δt (days)	*Triple conjunction*
−935	Apr. 22·57	Apr. 26·56	−3·99	2·9	no
−933	July 4·37	July 5·99	−1·62	1·8	yes
−931	Sept. 15·86	Sept. 17·41	−1·55	3·9	yes
−888	Apr. 12·99	Apr. 11·90	+1·09	4·5	yes
−886	June 20·16	June 19·85	+0·31	2·1	yes
−884	Sept. 5·19	Sept. 1·10	+4·09	2·4	no
+880	May 8·60	May 12·31	−3·71	3·0	no
882	July 19·04	July 21·99	−2·95	1·6	no
884	Oct. 2·00	Oct. 3·92	−1·92	3·8	yes
927	Apr. 30·25	Apr. 28·68	+1·57	4·7	yes
929	July 4·91	July 5·62	−0·71	1·8	yes
931	Sept. 22·00	Sept. 18·44	+3·56	2·3	no
2742	June 4·59	June 2·61	+1·98	4·7	yes
2744	Aug. 7·67	Aug. 9·41	−1·74	1·8	yes
2746	Oct. 26·76	Oct. 23·76	+3·00	2·2	no
2791	July 26·43	July 25·35	+1·08	2·7	yes

Note: Some borderline cases are also included. All twins include one triple conjunction with Mars near its perihelion. Δt denotes the extreme time difference between the opposition dates, where a triple conjunction is possible.

Saturn dates are taken from reference 6, while the others are extended versions of earlier calculations[5]. For Jupiter–Saturn, a periodicity of 973 years can be recognized from the table. Although for Jupiter–Saturn the shortest possible time between two TCs could be 20 years according to Table I, between AD 0 and 3000, one finds no shorter interval than 40 years. The TC of Jupiter–Saturn preceding the 332–333 case took place in 6 BC and has been discussed frequently in connection with the Star of Bethlehem.

In Table IV there are 31 cases for Mars–Jupiter, 40 for Mars–Saturn and 16 for Jupiter–Saturn. This agrees with the mean frequencies calculated earlier[4]. In 1503–04 there was a triple conjunction of Mars with Jupiter as well as with Saturn; this is illustrated in figure 2. Curiously, in the whole twenty-first century,

Table IV

Triple conjunctions in longitude of the bright outer planets for AD 0 to 3000

		Mars–Jupiter		
21–22	784	1170–1171	1836–1837	2599
164–165	884	1313–1314	1979–1980	2699–2700
307–308	927 } twin	1456–1457	2123	2742 } twin
354–355	929 }	1503–1504	2169–2170	2744 }
497–498	976	1646–1647	2313	2791
641	1027	1789–1790	2456	2842–2843
				2985–2986
		Mars–Saturn		
77	658	1264–1265	1779	2626–2627 } twin
177–178 } twin	756	1503–1504 } twin	1945–1946	2628–2629 }
179–180 }	822–823	1505–1506 }	2148–2149	2663
216	860–861	1542	2185 } twin	2761
380–381	1061–1062 } twin	1640	2187 }	2829–2830
418–419	1063–1064 }	1706–1707	2221	2866
519	1100	1742–1743 } twin	2319	
619–620 } twin	1198	1744–1745 }	2387–2388	
621–622 }				
		Jupiter–Saturn		
332–333	709–710	1305–1306	1940–1941	2279
411–412	967–968	1425	1980–1981	2655–2656
452	1007–1008	1682–1683	2238–2239	2794–2795
				2913–2914

Note: Two twins appear for Mars–Jupiter and seven for Mars–Saturn.

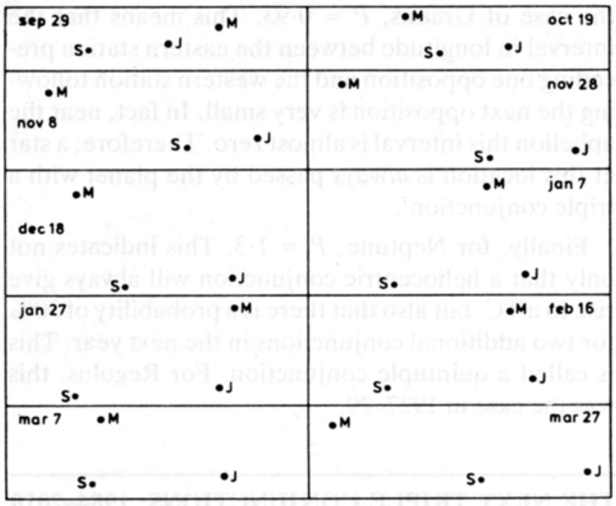

Figure 2. Positions of Mars (M), Jupiter (J) and Saturn (S) with 20 days interval in 1503–04. Ecliptic north is up. Each figure is 13° wide. The opposition dates of the planets were 1503 Dec. 26, Dec. 22 and Dec. 27, respectively.

no TC will take place: we have to wait until 2123 for the next triple conjunction between two bright outer planets!

The role of periodicities in the sequence of TCs is at best seen for Mars–Saturn. Here, the main periodicity is 442 years; indeed, all TCs are preceded or followed by another one after such a period. One may therefore construct series of TCs, all separated by 442 years, which may last for millenia. Such a series is 177, 619, 1061, 1503, 1945, 2387, 2829. In due course, a series ends or a new one starts. For instance, an old series of twins ended in 1506, while a new one began 239 years later in 1742.

Since Δt is relatively small near the perihelion of Mars, and because of the fact that this planet is more displaced with respect to Saturn after 442 years, all series near the perihelion of Mars are relatively short; the series 756, 1100, 1542 is one such example. In 2319, a new series of TCs will start near the perihelion of Mars. A prelude to this series occurred in 1877, when a TC in right ascension took place, but no TC in longitude.

Table V gives all TCs of the planets Jupiter to Neptune from AD 1900 to 2050. Since the relative synodic period of the far outer planets is so large, they have only a few triple conjunctions. The most extreme case is Uranus–Neptune, which has a mutual synodic period of 171 years. In 1993, the first conjunction between these planets will take place since the discovery of Neptune. As can be seen from Table V, this will be a triple conjunction. The conjunctions in longitude occur on the following dates:

1993 Feb. 2	Uranus 1° 05′ south of Neptune;
1993 Aug. 19	Uranus 1° 09′ south of Neptune;
1993 Oct. 25	Uranus 1° 06′ south of Neptune.

One synodic period earlier, in 1821, there was also a triple conjunction. On the next occasion, however, in 2165 there will be only a single conjunction of the two planets.

Table V

Triple conjunctions in longitude of the planets Jupiter to Neptune for AD 1900 to 2050

Jupiter–Uranus:	1927–1928 1954–1955 1968–1969 1983 2010–2011 2037–2038	*Jupiter–Neptune:*	1919–1920 1971 2009 2047–2048
Saturn–Uranus:	1988	*Saturn–Neptune:*	1952–1953 1989
Uranus–Neptune:	1993		

TRIPLE CONJUNCTIONS BETWEEN A BRIGHT PLANET AND A STAR

Triple conjunctions are by no means rare, however. In any century, many of them are produced between a bright planet and a bright star. Table VI gives all TCs of Mars, Jupiter and Saturn with the first-magnitude stars near the ecliptic for AD 1900 to 2050. Counting up we find there are 45 cases, that is one in about three years.

In principle, the circumstances for planet–star TCs are simpler than for planet–planet TCs. The reason for this is that such a TC will always happen in the same region of the ecliptic. In consequence, a TC with a given star will always take place around the same dates, and with the angular separation between the bodies comparable. For instance, for Jupiter–

Table VI

Triple conjunctions in longitude of the bright outer planets with the first-magnitude zodiacal stars, AD 1900 to 2050

Mars–Aldebaran	*Jupiter–Aldebaran*	*Saturn–Aldebaran*
1911–1912	1917–1918	1942–1943
1943–1944	1929–1930	2001–2002
1990–1991	2000–2001	
2022–2023	2012–2013	
Mars–Regulus	*Jupiter–Regulus*	*Saturn–Regulus*
1900–1901	1955–1956	1977–1978
1915–1916	1967–1968	2036–2037
1947–1948	2038–2039	
1979–1980	2050–2051	
1994–1995		
2026–2027		
Mars–Spica	*Jupiter–Spica*	*Saturn–Spica*
1920	1934	1952–1953
1935	1945–1946	2011–2012
1967	1957–1958	2041
2014	2028–2029	
2046	2040–2041	
Mars–Antares	*Jupiter–Antares*	*Saturn–Antares*
1969	1900	1956–1957
2048	1912	1986
	1983	2015–2016
	1995	2045

Regulus and for Saturn–Regulus the conjunctions are always quite close (less than one or two degrees), but for other combinations the planet remains always several degrees from the star at the triple conjunction.

From the table, periodicities can again be found. For Mars it is 79 years, and for Jupiter 83 years. Such periodicities can be determined by developing the sidereal period of the planet into a continued fraction (a similar method may be applied for planet–planet TCs). For Jupiter and Saturn, two TCs may also be separated by only one sidereal period of the planet. As far as Mars is concerned, the minimum time between two TCs is 15 years: after a mean time interval of 15·8 years an opposition of Mars takes place under very similar circumstances. So, every 15·8 years there may be a triple conjunction between Mars and a given star; for Jupiter and Saturn this is every 11·9 years and 29·5 years, respectively. Of course, not every opportunity of this kind results in a TC. The probability P for this equals $R/\Delta\lambda$, where R is the length of the retrograde path and $\Delta\lambda$ the difference in longitude of the planets between two successive favourable oppositions. For Mars, $P = 0{\cdot}13$ at a perihelic opposition and $P = 0{\cdot}58$ for an aphelic one. The mean frequency of a TC with Regulus (*e.g.* requiring an opposition of Mars near its aphelion) will be one in about 26 years. The variation of P also explains why TCs of Mars–Antares are less frequent than Mars–Spica. Incidentally, the difference in longitude between Spica and Antares is almost exactly equal to the difference in longitude of Mars after two successive oppositions in this part of the ecliptic. In consequence, a TC of Mars–Antares is *always* preceded by a TC of Mars–Spica, two years earlier. (See also Table VI.) Because of the difference in P for these stars, the reverse does not hold, however.

As to the other outer planets, it can be proved that P increases with distance from the Sun. For Jupiter, $P = 0{\cdot}3$ (indicating that only 30% of its heliocentric conjunctions with a star give rise to a triple conjunction, as seen from the Earth); for Saturn $P = 0{\cdot}5$. In the case of Uranus, $P = 0{\cdot}93$. This means that the interval in longitude between the eastern station preceding one opposition and the western station following the next opposition is very small. In fact, near the aphelion this interval is almost zero. Therefore, a star at this location is *always* passed by the planet with a triple conjunction[7].

Finally, for Neptune, $P = 1{\cdot}3$. This indicates not only that a heliocentric conjunction will always give rise to a TC, but also that there is a probability of 30% for two additional conjunctions in the next year. This is called a quintuple conjunction. For Regulus, this was the case in 1927–29.

THE NEXT TRIPLE CONJUNCTIONS: 1984–2010

In the next 140 years, no triple conjunction of two bright outer planets will take place. Neglecting the TCs of Venus–Jupiter, the only spectacular events for the near future are planet–star TCs. These cases are listed below in Table VII, which has been extracted from the other tables in this paper.

Table VII

Future triple conjunctions (1984–2010) in longitude of a bright outer planet with a bright object

Planets	*Aldebaran*	*Regulus*	*Antares*
Mars	1990–1991	1994–1995	–
Jupiter	2000–2001	–	1995
Saturn	2001–2002	–	1986

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