# Identification of odd-radius halo arcs and of $44^{\circ} / 46^{\circ}$ parhelia by their inner-edge polarization 

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#### Abstract

Birefringence of ice causes the inner edges of refraction halos to be polarized. The direction of this polarization relates directly to the projection of the crystal main axis onto the sky. This implies that the inner-edge polarization can serve as an observational diagnostic for determining the actual nature of a halo arc if two competing explanations exist. The direction and the visibility of the inner-edge polarization of arcs and circular halos arising from usual ice crystals and from ice crystals with pyramidal ends are calculated. It is found that the observation of inner-edge polarization can be decisive for the identification of a spot that might be either a $44^{\circ}$ parhelion or a $46^{\circ}$ parhelion, of an arc that might be either a $22^{\circ}$ sunvex Parry arc or a $20^{\circ}$ Parroid arc arising from plate-oriented pyramidal crystals, and of an arc that might be either a $22^{\circ}$ suncave Parry arc or a $23^{\circ}$ Parroid arc from plate-oriented pyramidal crystals. (With a Parroid arc, a halo is that which arises from an ice wedge made up of two faces of a crystal that rotates about a vertically oriented spin axis, and the edge of the wedge is perpendicular to this spin axis.) Polarization properties of other rare arcs are discussed. Practical hints are given for observing visually the inner-edge polarization of halos. © 1998 Optical Society of America

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## 1. Introduction

A verdict of reliable or unreliable of a report of a strange halo sighting often depends on the possibility of supplementing the observation with a plausible mechanism of formation. Failure to do so often brings in a qualification of unreliable, and then there is hardly ever a way back. The exploration of computer simulation techniques ${ }^{1}$ has emerged that with hindsight too many old observations have been dismissed on these grounds. In particular, the brute force ray-tracing programs, ${ }^{2-4}$ fed by well-established crystal orientation modes, have actually been able to simulate correctly a number of traditionally dismissed halo observations, which have led to true reincarnations of the credibility of some early observers. Tape ${ }^{4}$ comments rightly on the situation: "In general, we perhaps too easily dismiss the work of previous generations." In this perspective, it is interesting to mention the attitude of Alfred Wegener himself in the discussions about his controversial continental drift theory, in which he considered the

[^0]lack of a plausible driving force as being no serious counterargument at all to this theory. Whenever that point was raised during one of his many lectures, he invariably responded with "Die Kräfte wirden sich wohl finden" ("Don't bother, one day the forces will show up")! ${ }^{5}$

Despite the progress of the simulation techniques, there are still halos that managed to escape from complete understanding. A notorious example is the $44^{\circ} / 46^{\circ}$ parhelion, whose existence has been unambiguously proved by a photograph. ${ }^{6}$ Two competing explanations are proposed for this halo ${ }^{7-9}$ : double scattering by plate-oriented crystals (main crystal axis vertical; the $44^{\circ}$ parhelion) and single scattering by alternate Parry-oriented crystals (main crystal axis horizontal and two prism faces vertical; the $46^{\circ}$ parhelion). Both explanations call for exceptional circumstances, but as yet there are no firm grounds for dismissing a priori one of them. Another type of ambiguity has actually been recognized quite recently. ${ }^{4}$ Antarctic observations, supported by computer simulations, have shown that plateoriented crystals with pyramidal ends (Fig. 1) can be present in sufficient quantities to produce bright arcs associated with odd-radius halos. Some of these arcs turn out to resemble the $22^{\circ}$ Parry arcs so closely that they can be easily be mistaken for them.

A traditional strategy to decide what mechanism is actually acting is to look for associated halos that

| Entry and <br> exit faces | Circular halo <br> radius | Name of the <br> circular halo |
| ---: | :---: | :--- |
| $3-16$ | $9.0^{\circ}$ | $9^{\circ}$ halo |
| $13-25$ | $18.3^{\circ}$ | $18^{\circ}$ halo |
| $13-16$ | $19.9^{\circ}$ | $20^{\circ}$ halo |
| $1-23$ | $22.9^{\circ}$ | $23^{\circ}$ halo |
| $3-15$ | $23.8^{\circ}$ | $24^{\circ}$ halo |
| $13-15$ | $34.9^{\circ}$ | $35^{\circ}$ halo |



Fig. 1. Pyramidal crystal, the numbering of the faces and the angles of halos of which at least one of the two refracting faces is a pyramidal face. The bottom face has face number 2 . The crystal main axis ( $C$ axis) coincides with the crystal optic axis. The figure is taken with permission from Ref. 4.
should occur in either mechanism; another approach is to take accurate measurements of the arc's distance to the Sun. In this paper a guideline for a third diagnostic is given, i.e., inner-edge polarimetry. The direction of the polarization of the inner edge of a halo or arc relates directly to the projection onto the sky of the main axis ( $C$ axis) of the halo-generating crystals. Hence observation of this polarization direction provides a direct clue to the preferential orientation of the crystals; if two competing orientations are proposed, such an observation can be decisive in finding the correct one. In this way, inner-edge polarimetry may be a tool to resolve some of the mysteries mentioned above.

After an introductory section about halo naming, the inner-edge polarization of (odd-radius) circular halos and that of the associated arcs and parhelia are discussed; candidates are selected for which polarimetric observations with the eye of the inner edge may be successful and decisive for finding the correct halo mechanism. The paper ends with observational hints.

## 2. Parroid Arcs and Side Arcs

For the nomenclature of odd-radius arcs, a simple pragmatic scheme that aims at the introduction of only as few as possible new names is applied. The basis of the scheme is the classification ${ }^{10}$ of arcs according to the orientation of the halo-generating refracting ice wedges with respect to a spin axis fixed in the wedge (this axis is denoted by the vector $P$ in Ref. 10); the wedge is free to rotate about this spin axis. Two categories are of relevance: spin axis vertical and spin axis horizontal. In the latter case the spin axis is free to rotate about the vertical. Wedges in plate-oriented ice crystals or (alternate) Parryoriented crystals fit into the first category; wedges in column-oriented or in Lowitz-oriented crystals fit into the second category. For a full account of all possible halos, see Ref. 10.

In accordance with the usual convention, ${ }^{7,8}$ we call arcs parhelia when they arise from wedges whose
refracting edge is parallel with the spin axis and the spin axis is vertical. A parhelion lies entirely on the parhelic circle and contacts the associated circular halo (that is, the halo arising from randomly oriented wedges with the same wedge angle) at a solar elevation of zero. Arcs due to wedges with their refraction edge again parallel with the spin axis but now with the spin axis horizontal are called tangent arcs, which also follows tradition. ${ }^{7,8}$ Tangent arcs are symmetrical with respect to the solar vertical and have for any solar elevation a point of contact with the associated circular halo; the points of contact are in the solar vertical. Examples among the familiar halos are the $22^{\circ}$ upper and lower tangent arcs (or the circumscribed halo) and the $46^{\circ}$ tangent arcs, also known as Galle's arcs.

Next to these arcs we need to define a new class of arcs, called here Parroid arcs. This class of Parroid arcs represents a generalization of the Parry arcs. Parroid arcs arise from wedges in which the spin axis is vertically oriented and the refracting edge is perpendicular to this axis. For a given Parroid arc, the inclinations of the entry and the exit faces with respect to the vertical have fixed values. A Parroid arc is symmetrical with respect to the solar vertical and contacts its associated circular halo at one solar elevation; the contact point is in the solar vertical. In Ref. 10, Parroid arcs are represented by point halos with halo pole $P_{u}$ on the Bravais equator, $y=0$. Familiar examples of arcs that fit into the class of Parroid arcs are the $22^{\circ}$ (alternate) Parry arcs and the $46^{\circ}$ circumzenithal arc. The latter example illustrates that the name Parroid arc is not meant to suggest an exclusive relation with Parry-orientated crystals: Plate-oriented crystals can also give rise to Parroid arcs, including odd-radius ones. Figure 2 illustrates this for $20^{\circ}$ and $23^{\circ}$ Parroid arcs.

The remaining ${ }^{10}$ possible halo arcs have in common that their point(s) of contact with their associated circular halo is never in the solar vertical, except (sometimes) for spin axis horizontal, but even then it occurs at only one solar elevation. In the context of this paper it is of no use to introduce a refined naming system for these arcs. Instead, they are all referred to as side arcs, regardless of whether the orientation of the spin axis is horizontal or vertical. Hence the name side arcs refers to a broad category of arcs, as it consists of all possible halo arcs, with the exclusion of only parhelia, tangent arcs, and Parroid arcs. Examples of arcs that fit into the class of side arcs from vertically oriented spin axes are the $46^{\circ}$ Parry supralateral arcs, ${ }^{4}$ nowadays also called Tape arcs; examples of side arcs from horizontally oriented spin axes are the $22^{\circ}$ Lowitz arcs and the $46^{\circ}$ supralateral and infralateral arcs.

## 3. Direction and Visibility of Inner-Edge Halo Polarization

The theory of the inner-edge polarization of refraction halos has been described elsewhere ${ }^{11,12}$ and is applied explicitly to $22^{\circ}$ and $46^{\circ}$ halo forms. In this section, it is further worked out for halos arising from crystals

## $20^{\circ}$ Parroid arc


$20^{\circ}$ Parroid arc

$23^{\circ}$ Parroid arc


Fig. 2. Examples of formation of a Parroid arc in pyramidal crystals of different orientations. By definition, a Parroid arc is an arc that arises from refraction through wedges that spin about a vertical axis and the refracting edge of the wedge is perpendicular to this axis (hence horizontal). The two upper diagrams show a light path for a $20^{\circ}$ Parroid arc in Parry-oriented crystals (main crystal axis and two prism faces horizontal; left diagram) and in plateoriented crystals (main crystal axis vertical; right diagram). The two lower diagrams give examples for $23^{\circ}$ Parroid arcs. Each Parroid arc has its own shape. In the depicted crystals, ray paths exist for other $20^{\circ}$ Parroid arcs or other $23^{\circ}$ Parroid arcs and also for Parroid arcs associated with circular halos of other radii. These possibilities are not shown here.
with pyramidal ends. Only polarization effects due to birefringence are considered; in the perspective of the current application, changes in polarization due to Fresnel losses at the entrance and the exit faces can be neglected.

The inner-edge polarization of halos arises because of the birefringence of ice. This results in a polarization dependence of the index of refraction. Unpolarized light that enters a crystal splits up into two fully polarized rays. Halos due to ordinary refracted light rays appear slightly closer to the Sun than do halos due to extraordinary refracted rays. These two components of the halo are orthogonally polarized; the shift between them is denoted by $\Delta \theta_{h}$.

In geometric optics and in the absence of halobroadening effects such as solar-disk smearing, the inner boundary of each polarized halo component consists of a jump of the halo radiance from zero to a finite value. ${ }^{11,12}$ Hence, between the onsets of the two components, only halo light due to ordinary refracted light is visible (see Fig. 2 of Ref. 12); generally this light is strongly polarized. We call this narrow peak of polarized light at the inner edge of a halo the (geometric) birefringence peak of the halo; the width of this peak is equal to the mutual shift $\Delta \theta_{h}$ of the two polarized halo components.

The birefringence of ice $n_{e}-n_{o}$ equals ${ }^{13} 0.0014$. The ordinary refracted ray is subject to the ordinary refractive index $n_{o}=1.31$; the wave normal of the
extraordinary refracted ray is subject to a slightly higher index of refraction $n_{\text {eff }}$, where $n_{o}<n_{\text {eff }} \leq n_{e}$. The difference in refractive index of the two polarized rays $\Delta n=n_{\text {eff }}-n_{o}$ for ice can be found from the refraction law for birefringent materials, ${ }^{14}$ and can be simplified because of the weak birefringence to

$$
\begin{equation*}
\Delta n=n_{\mathrm{eff}}-n_{o}=\left(n_{e}-n_{o}\right) \sin ^{2} \gamma, \tag{1}
\end{equation*}
$$

where $\gamma$ is the angle between the light ray inside the crystal and the crystal optic axis. For ice, the optic axis coincides with the main crystal axis.
The shift $\Delta \theta_{h}$ of the two polarized halo components of circular halos can be found from the minimum deviation formula for a prism and is

$$
\begin{equation*}
\Delta \theta_{h}=\frac{180^{\circ}}{\pi} \frac{2 \sin \left(\theta_{h} / 2\right)}{n_{o} \cos \left(\theta_{h} / 2\right)-1} \Delta n \tag{2}
\end{equation*}
$$

where $\theta_{h}$ is the halo scattering angle and $\Delta \theta_{h}$ is expressed in degrees.

The direction of polarization of the extraordinary refracted ray is in the plane formed by the light ray inside the crystal and the crystal optic axis; that of the ordinary refracted ray is perpendicular to such a plane. The angle $\psi$ defines the tilt of the refracting edge with respect to this plane (see Fig. 3). A refracting ice wedge that contributes to the radiance of the inner edge of a circular halo has its refracting edge perpendicular to the scattering plane. From the viewpoint of an observer looking to a spot on the circular halo's inner locus, $\psi$ represents the tilt with respect to that inner locus of the projection onto the sky of the optic axis (=main axis) of an individual crystal that contributes to the halo radiance. The circular halo's inner locus is perpendicular to the scattering plane, and the polarization of ordinary refracted light is perpendicular to the projection of the optic axis onto the sky. Therefore $\psi$ is also the tilt with respect to the scattering plane of the polarization of light that emerges from an individual crystal that contributes to the halo inner-edge radiance.

For circular halos, as well as for tangent arcs and Parroid arcs in their regions that are closest to the solar vertical, there are two crystal configurations that contribute to the inner halo radiance, one with tilt $+\psi$ and one with tilt $-\psi$. For circular halos from wedges with $\psi=k .90^{\circ}$ ( $k$ is an integer) both contributions have the same polarization and therefore the geometric birefringence peak is completely polarized. However, for circular halos arising from wedges with $\psi \neq k .90^{\circ}$, thus for the $24^{\circ}$ and $35^{\circ}$ halos ( $\psi=28^{\circ}$ and $47^{\circ}$, respectively), halo light from the two configurations has a different direction of polarization, and hence the two contributions together cause the halo geometric birefringence peak to be only partially polarized, ${ }^{11}$ with a direction of the polarization that is, just as for the $\psi=k .90^{\circ}$ circular halos, either in the scattering plane or perpendicular to this. The degree of polarization $P$ of the geometric halo birefringence peak of a circular halo is given by ${ }^{11}$

$$
\begin{equation*}
P=\cos (2 \psi) . \tag{3}
\end{equation*}
$$



Fig. 3. Minimum-deviation ray passing though a halo-generating ice wedge consisting of two of the crystal faces of Fig. 1. The scattering plane is perpendicular to the refracting edge and hence in this view is horizontal. The angle $\gamma$ determines the refractive index of the extraordinary refracted rays [Eq. (1)]. The direction of polarization of the extraordinary refracted rays is in the plane formed by the light ray and the crystal main axis; that of ordinary refracted rays (not shown) is perpendicular to such a plane. The latter rays make up the halo inner edge. For a circular halo, the angle $\psi$ defines the tilt with respect to the scattering plane of the polarization of light of an individual crystal wedge that contributes to the halo inner-edge radiance.

The sign of $P$ determines the direction of the inneredge polarization: If $P$ is positive, it is in the plane of scattering; $P$ is negative implies that the direction is perpendicular to the plane of scattering.

For a side arc there is only one crystal configuration left to contribute to the radiance on certain spot on the arc's inner edge. Therefore the geometric birefringence peaks of side arcs associated with $\psi \neq$ $k .90^{\circ}$ circular halos (the $24^{\circ}$ and $35^{\circ}$ ones) remain fully polarized while at the same time the side arc's inner-edge polarization is tilted (by approximately $\psi$ ) with respect to the scattering plane. This results in noticeable differences in the states of inner-edge polarization of $24^{\circ}$ or $35^{\circ}$ side arcs with respect to their associated circular halo. This is discussed further in Section 4.

All other arcs, including the side arcs to all $\psi=$ $k .90^{\circ}$ circular halos (radii $9^{\circ}, 18^{\circ}, 20^{\circ}, 22^{\circ}, 23^{\circ}$, and $46^{\circ}$ ), have (roughly) the same direction of inner-edge polarization as that of their associated circular halos. This statement is completely true at a point of contact with the circular halo. A parhelion contacts its associated circular halo at zero solar elevation. For other solar elevations, the direction of polarization of the parhelion with respect to the vertical persists. Consequently the inner-edge parhelion polarization is either horizontal or vertical. Plate orientation implies vertically oriented crystal main axes and can make a $22^{\circ}$ parhelion (by single scattering) and perhaps a $44^{\circ}$ parhelion (by double scattering7-9); both should have a horizontal inner-edge polarization.

The only other crystal orientation capable of producing parhelia is the still-unproved alternate Parry orientation; this orientation would generate parhelia to the $9^{\circ}, 20^{\circ}, 23^{\circ}$, and $46^{\circ}$ halos. The inner edges of these four types of parhelia should be vertically polarized.

The visibility of the inner-edge polarization of a circular halo or an arc is proportional to its shift $\Delta \theta_{h}$, to the absolute degree of polarization of the geometric birefringence peak $|P|$, and to the steepness of its radiance distribution near its edge. ${ }^{11}$ The steepness relates to the halo broadening and depends on many variables, including the effective slit width of the crystals and the departure from an idealized preferential crystal orientation. ${ }^{12}$ Therefore we define a visibility Vis relative to the $22^{\circ}$ halo types, in which the effect of broadening is not taken into account. This visibility Vis is given by

$$
\begin{equation*}
\text { Vis }=|P| \Delta \theta_{h} / \Delta \theta_{h}\left(22^{\circ}\right)=|P| \Delta \theta_{h} / 0.106^{\circ} . \tag{4}
\end{equation*}
$$

Table 1 shows $\gamma, \psi, \Delta \theta_{h}$, the direction of inner-edge polarization, and its visibility Vis calculated from Eqs. (3) and (4) for circular halos that may arise from pyramidal crystals. Arcs associated with these circular halos should have (approximately) the same Vis near their contact points as the associated circular halo, with the exception of the $24^{\circ}$ and the $35^{\circ}$ side arcs. The visibility Vis of the inner-edge polarization of the latter two should be calculated from Eq. (4) by putting $P=1$.

In Table 1, halos are marked whose inner-edge polarization can be visible to the eye. Here the threshold of visibility is put at Vis $=0.3$. Theoretically this value may be somewhat lower, as the $22^{\circ}$ halo shift $\Delta \theta_{h}=0.106^{\circ}$ exceeds the angular resolving power of an eye by a factor of 8 . Therefore inneredge polarization with Vis $<0.12$ can be considered to be essentially invisible to the human eye.

A special class of arcs is formed by the Parry and the Parroid arcs. Except for the solar elevation at which such an arc contacts its associated circular halo, the ray path through the ice wedge that causes the arc's radiance in the solar vertical is not a ray of minimum deviation. Consequently the value of $\Delta \theta_{h}$ cannot be calculated from Eq. (2) and is dependent on solar elevation. For the $22^{\circ}$ Parry arcs at their points straight over or straight under the Sun, $\Delta \theta_{h}$ is proportional to the width of color band of the arc. This is true because the light rays cross the optic axis always at a right angle $\left(\gamma=90^{\circ}\right)$ and hence $\Delta n$ is independent of solar elevation [see Eq. (1)]. The latter is not the case for the $20^{\circ}$ and $23^{\circ}$ Parroids that arise from plate-oriented pyramidal crystals. Particularly for the $23^{\circ}$ Parroid, $\sin ^{2} \gamma$ and hence $\Delta n$ depend so strongly on solar elevation that it modifies the dependence of $\Delta \theta_{h}$ and Vis on solar elevation considerably. This is demonstrated in Fig. 4.

Table 1. Inner-Edge Polarization of Circular Halos

| Halo | Faces | Angle $\gamma$ between Optic Axis and Ray (deg) | Tilt $\psi$ of Polarization Direction of an Ordinary Refracted Ray with Respect to Scattering Plane (deg) | Shift $\Delta \theta_{h}$ of the Polarized Halo Components (deg) | Direction Inner-Edge <br> Polarization with <br> Respect to Scattering <br> Plane | Visibility of Inner-Edge Polarization $\left(22^{\circ}\right.$ halo $\left.=1\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $9^{\circ}$ | 3-16 | 76 | 90 | 0.04 | Perpendicular | $0.4{ }^{\text {a }}$ |
| $18^{\circ}$ | 13-25 | 58 | 0 | 0.06 | Parallel | $0.6{ }^{\text {a }}$ |
| $20^{\circ}$ | 13-16 | 90 | 90 | 0.10 | Perpendicular | $0.9^{a}$ |
| $22^{\circ}$ | 3-5 | 90 | 0 | 0.11 | Parallel | $1.0^{\text {a }}$ |
| $23^{\circ}$ | 1-23 | 31 | 90 | 0.03 | Perpendicular | 0.3 |
| $24^{\circ}$ | 3-15 | 74 | 28 | 0.11 | Parallel | $0.6{ }^{a}$ |
| $35^{\circ}$ | 13-15 | 90 | 47 | 0.19 | Perpendicular | 0.1 |
| $46^{\circ}$ | 1-3 | 45 | 90 | 0.15 | Perpendicular | $1.4^{a}$ |

${ }^{a}$ Inner-edge polarization may be visible to the eye.

## 4. Rules of Thumb for Discriminating Arcs by their Inner-Edge Polarization

The polarization direction of the inner edge of a given arc can be easily inferred from a known position of the crystal main axis of the halo-generating crystals in the sky. If a possible change of the polarization direction at the exit face of the crystal is neglected, the polarization of the least-refracted ray is perpendicular to the main axis of the crystals that scatter the halo light. Hence the inner-edge halo polarization is perpendicular to the projection onto the sky of the main axes of the halo-generating crystals. Plate-oriented crystals have the axis vertical; singly oriented columns or doubly oriented columns (Parry and alternate Parry orientations) have the axis horizontal. To find the inner-edge polarization of an arc, one has to consider how the realization of the ray path of the inner-edge radiance freezes the projection of the crystal main axis onto the sky down into one or two solutions.

Rules of thumb for the inner-edge polarization are

- Arcs from plate-oriented crystals are horizontally polarized at their whole inner boundary. This holds for all such arcs, including the side arcs.
- With the exception of the $24^{\circ}$ and the $35^{\circ}$ side arcs, all arcs from singly or doubly oriented columns and from spinning crystals have in their points of contact with the associated circular halo the same direction of polarization at their inner edge as that of the circular halo. If the arc has no point of contact, the regions at the arc and at the circular halo that are closest to each other have in general the same directon of inner-edge polarization.
- Side arcs associated with the $24^{\circ}$ or the $35^{\circ}$ halo have a direction of inner-edge polarization that is tilted with respect to that of the associated circular halo. This property holds for both types of side arc.
- Parhelia are either horizontally or vertically polarized at their inner boundaries. Only plate orientation can result in the former.

The first and the third rules indicate that the direction in which a $24^{\circ}$ or a $35^{\circ}$ side arc moves when
viewed through a rotating polarizer (the line arcSun) corresponds to neither the polarizer's position at maximum transmission of the arc's inner limb radiance nor to its position at minimum transmission of the arc's inner limb radiance. As mentioned above, another specific property of the $24^{\circ}$ and the $35^{\circ}$ side arcs is the high degree of inner-edge polarization compared with that of their associated circular halos.

Taken together, the rules indicate that arcs due to plate orientation can often be discriminated from arcs due to doubly oriented columns by their inner-edge polarization, although not in all cases. An illustrative counterexample is the circumzenithal arc, whose inner edge right over the Sun remains horizontally polarized when the arc arises from Parry-oriented crystals instead of from plate-oriented crystals. On the other hand, many cases remain in which the determination of the inner-edge polarization can be decisive. Table 2 gives a list of pairs of arcs resembling


Fig. 4. Angular displacement $\Delta \theta_{h}$, as seen through a rotating polarizer, of the inner edges of the upper sunvex and suncave Parry arcs and of their closely resembling counterparts, the upper $20^{\circ}$ and $23^{\circ}$ Parroid arcs arising from plate-oriented pyramidal crystals. The right axis represents the visibility Vis of the arc's inneredge polarization relative to that of the $22^{\circ}$ halo. The light paths of the arcs are depicted schematically.

Table 2. Some Pairs of Halo Arcs that may be Mistaken for Each Other, Whose Real Nature may be Determined by their Inner-Edge Polarization

| Halo Arc | Crystal Orientation | Shift $\Delta \theta_{h}$ of the Polarized Halo Components (deg) | Inner-Edge Polarization Direction | Visibility of Inner-Edge Polarization |
| :---: | :---: | :---: | :---: | :---: |
| $23^{\circ}$ Parroid arcs | Plate | $0.02-0.07^{a}$ | Horizontal | $0.2-0.6^{a}$ |
| $22^{\circ}$ suncave Parry arcs | Parry | 0.11 | Vertical | 1 |
| $20^{\circ}$ Parroid arcs | Plate | $0.14{ }^{\text {b }}$ | Horizontal | $1.3{ }^{\text {b }}$ |
| $22^{\circ}$ sunvex Parry arcs | Parry | $0.19{ }^{\text {b }}$ | Vertical | $1.8{ }^{\text {b }}$ |
| $44^{\circ}$ parhelia | Plate, double scattering | 0.21 | Horizontal | $1.4{ }^{\text {c }}$ |
| $46^{\circ}$ parhelia | Alternate Parry | 0.15 | Vertical | 1.4 |

${ }^{a}$ In case of the upper arc the higher values are for low $\left(<30^{\circ}\right)$ solar elevation (see Fig. 4).
${ }^{b}$ Value for the upper arc at solar elevation $10^{\circ}$ (see Fig. 4).
${ }^{c}$ Takes into account the excess broadening due to double scattering.
each other closely, for which the inner-edge polarization is a diagnostic to determine their true nature.

## 5. Unobserved and Dubious Arcs

Arcs from plate-oriented pyramidal crystals do occur in nature; the brilliant Sturm photograph in Tape's book ${ }^{4}$ clearly shows, among other arcs, the lower $9^{\circ}$ Parroid arc, the upper $23^{\circ}$ Parroid arc, and the $24^{\circ}$ side arcs arising from plate orientation. Arcs from singly oriented column crystals with pyramidal ends (main axis horizontal) have also occasionally been observed. ${ }^{10,15}$ Odd-radius arcs due to Parryoriented crystals should be much rarer and are never identified. One can speculate about the occurrence of three other orientation modes of pyramidal crystals: spinning, the alternate Parry orientation, and perhaps the hypothetical pyramidal-Parry orientation (one pyramidal face horizontal).

All orientation modes together lead to an almost countless number of different arcs. Because of the dubious character of the existence of many of them, it is of no use to analyze them all in detail. Perhaps the least unlikely unobserved odd-radius arcs are the Parroid arcs arising from Parry-oriented crystals. Table 3 lists their properties ( $9^{\circ}$ and $46^{\circ}$ arcs are not
included). In combination with Table 2 it shows all Parroid arcs from either plate orientation or Parry orientation that may be mistakenly identified as $22^{\circ}$ (alternate) Parry arcs.

Perhaps the most intriguing aspect of Table 3 is that the $22^{\circ}$ alternate Parry arcs, which do not require unusually shaped crystals but a still unproved crystal orientation mode, are difficult to distinguish from the $20^{\circ}$ and the $23^{\circ}$ Parroid arcs arising from the rare pyramidal crystals in the rare but existing Parry orientation. As in the Table 2 cases, the observation of the inner-edge polarization provides a means to decide about its actual nature if an alleged $22^{\circ}$ alternate Parry arc appears.

## 6. Observational Hints and Practical Applications

The observation technique of inner-edge halo polarization is simple, but some practice on the usual halos (e.g., $22^{\circ}$ parhelia) is advisable. Simple sunglasses with polarizing lenses can be your instrument; their lenses have the direction of polarization (maximum transmission) vertical. First check whether the lenses are actually polarizers by observing the glare on a water surface while rotating the sunglasses. If

Table 3. Parroid Arcs from Parry-Oriented Crystals with Pyramidal Ends Compared with $\mathbf{2 2}^{\circ}$ (Alternate) Parry Arcs

| Halo Angle <br> (deg) | Number <br> of Types |  |  |
| :---: | :---: | :--- | :---: |
| 18 | 2 | Shapes of the Parroid Arcs | Inner-Edge <br> Polarization Direction ${ }^{b}$ |
| 20 | 1 | Like the sunvex $22^{\circ}$ Parry arcs | Vertical |
| 23 | 3 | Like the high-Sun upper alternate $22^{\circ}$ Parry arc | Horizontal |
| 24 | 2 | Like the remaining alternate $22^{\circ}$ Parry arcs | Horizontal |
|  | Like the suncave $22^{\circ}$ Parry arcs | Vertical |  |
| 22 | 4 | $22^{\circ}$ Parry arcs | Vertical |
| 22 | 4 | Alternate $22^{\circ}$ Parry arcs ${ }^{d}$ | Vertical |

[^1]the glare changes intensity during the rotation the lens is indeed a polarizer; the minimum intensity should be reached with the sunglasses in the normal horizontal wearing position, which verifies that the polarizers are correctly mounted.

Then wait for a bright $22^{\circ}$ parhelion. When it shows up, observe it through the polarizing sunglasses. Concentrate your view on the red parhelion inner limb. Rotate the glasses quickly from horizontal to vertical, concentrate on the inner limb, and rotate the glasses quickly back. The inner limb will shift its position during the rotation; you will note that the limb position is closest to the Sun when the sunglasses are held vertically. Hence the polarization direction of the inner limb is horizontal. After a while, the detection of the inner limb polarization becomes increasingly easy; one even starts to wonder how this effect could ever escape observation.

The next step is to extend your observation to other $22^{\circ}$ halo forms. In general you will find the observation more difficult in fuzzy halos or in the circular $22^{\circ}$ halo, whose relative fuzziness is actually an intrinsic property of the halo itself. ${ }^{16,17}$ Then try the $46^{\circ}$ circumzenithal arc to experience that for halos far from the Sun the inner-edge polarization is not so easy to observe, as it interferes with the polarization of the remainder of the halo as well as with that of the blue sky; however, even here the inner-edge polarization is noticeable with some practice.

After having done the exercises, one has to wait for a suitable candidate (Tables 2 and 3) to do the real observation. The most important point is to remember the position of the glasses when the inner limb is closest to the Sun. The observations are probably easier for arcs than for the circular halos. My impression is that inner limb polarizations with visibility Vis of 0.3 can still be observable.

I emphasize that the determination of inner-edge polarization is just an additional technique for halo identification that can never fully replace classical methods like the determination of the distance to the Sun or the detection of related halo phenomena. But in some cases, as in those of Table 2, doubt may persist, and then the direction of the inner-edge polarization is decisive. Of course, the arcs and spots listed in Table 2 are extremely rare, but perhaps you are the next person to observe and photograph an
alleged $23^{\circ}$ Parroid arc, an alleged $20^{\circ}$ Parroid arc, or a $44^{\circ} / 46^{\circ}$ parhelion; perhaps you may even be the first to see one of the Table 3 arcs. So be sure to keep your polarizer always with you!

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[^1]:    ${ }^{a}$ Refers to the number of different arcs that may arise from hexagonal pyramidal crystals for positive solar elevation. For the $22^{\circ}$ Parry arcs the four types are the upper-lower sunvex Parry arcs and the upper-lower suncave Parry arcs.
    ${ }^{b}$ The visibility of the polarization of a $18^{\circ}, 20^{\circ}, 24^{\circ}$ Parroid arc is 0.6-0.9 of that of its resembling $22^{\circ}$ (alternate) Parry arc (see also Table 1). The visibility of the polarization of a $23^{\circ}$ Parroid arc is at least two times smaller than that of its resembling $22^{\circ}$ alternate Parry arc (compare Fig. 4).
    ${ }^{c}$ Arises from Parry-oriented crystals.
    ${ }^{d}$ Arises from alternate Parry-oriented crystals.

